Dynamical Dark Matter
Collider Signatures and Direct Detection

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Work done in collaboration
with Keith Dienes:
[arXiv:1106.4546]
[arXiv:1107.0721]
[arXiv:1203.1923]
[arXiv:1204.4183] also with Shufang Su
[arXiv:1207.xxxx] also with Jason Kumar
Dynamical Dark Matter (DDM)

- The dark-matter candidate is an **ensemble** consisting of a vast number of constituent particle species whose collective behavior transcends that of traditional dark-matter candidates.

- Dark-matter stability is not a requirement; rather, the individual abundances of the constituents are **balanced against decay rates** across the ensemble in manner consistent with observational limits.

- Cosmological quantities like the total dark-matter relic abundance, the composition of the dark-matter ensemble, and even the dark-matter equation of state exhibit a **non-trivial time-dependence** beyond that associated with the expansion of the universe.

**Keith's talk:**
- General features of the DDM framework
- Characterizing the cosmology of DDM model
- An explicit realization of the DDM framework which satisfies all applicable constraints

**This talk:**
- Phenomenological consequences of DDM ensembles and methods of distinguishing them from traditional DM candidates experimentally.
Overview:

In many DDM models, constituent fields in the DDM ensemble can be produced alongside SM particles by the decays of additional heavy fields.

Evidence of a DDM ensemble can be ascertained in characteristic features imprinted on the invariant-mass distributions of these SM particles.

In this talk, I'll discuss methods for distinguishing DDM ensembles at the LHC...

K. Dienes, S. Su, BT [arXiv:1204.4183]

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and direct-detection experiments.

K. Dienes, J. Kumar, BT [arXiv:1205.xxxx]

- DDM ensembles can also give rise to distinctive features in recoil-energy spectra.

These are just two examples which illustrate that DDM ensembles give rise to observable effects which can serve to distinguish them from traditional DM candidates.
Part I: Distinguishing DDM at the LHC
Searching for Signs of DDM at the LHC

- In a wide variety of DM models, dark-sector fields can be produced via the decays of some heavy “parent particle” $\psi$.

- Strongly interacting $\psi$ can be produced copiously at the LHC. $SU(3)_c$ invariance requires that such $\psi$ decay to final states including not only dark-sector fields, but SM quarks and gluons as well.

- In such scenarios, the initial signals of dark matter will generically appear at the LHC in channels involving jets and $E_T$.

Further information about the dark sector or particles can also be gleaned from examining the kinematic distributions of visible particles produced alongside the DM particles.

As we shall see, such information can be used to distinguish DDM ensembles from traditional DM candidates on the basis of LHC data.
Traditional DM Candidates

- Let’s begin by considering a dark sector which consists of a traditional dark-matter candidate $\chi$ — a **stable** particle with a mass $m_\chi$.

- For concreteness, consider the case in which $\psi$ decays primarily via the **three-body** process $\psi \rightarrow jj\chi$ (no on-shell intermediary).

- Invariant-mass distributions for such decays manifest a **characteristic shape**.

- Different coupling structures between $\psi, \chi$, and the SM quark and gluon fields, different representations for $\psi$, etc. have only a small effect on the distribution.

- $m_{jj}$ distributions characterized by the presence of a **mass “edge”** at the kinematic endpoint: $m_{jj} \leq m_\psi - m_\chi$.
Parent Particles and DDM Daughters

In general, the constituent particles $\chi_n$ in a DDM ensemble and other fields in the theory through some set of effective operators $O_n^{(\alpha)}$:

$$\mathcal{L}_{\text{eff}} = \sum\sum_{\alpha n=0}^{N} \frac{c_{n\alpha}}{\Lambda^{d_{\alpha}-4}} O_n^{(\alpha)} + \ldots$$

As an example, consider a theory in which the masses and coupling coefficients of the $\chi_n$ scale as follows:

$$c_{n\alpha} = c_{0\alpha} \left( \frac{m_n}{m_0} \right)^{\gamma_{\alpha}}$$

$$m_n = m_0 + n^\delta \Delta m$$

$\delta$: scaling index for the density of states  
$\gamma_{\alpha}$: scaling indices for couplings  
$m_0$: mass of lightest constituent  
$\Delta m$: mass-splitting parameter  
Including coupling between $\psi$ and the dark-sector fields $\chi_n$. 
**Parent-Particle Branching Fractions**

Once again, let's consider the simplest non-trivial case in which $\psi$ couples to each of the $\chi_n$ via a four-body interaction, e.g.:

$$\mathcal{L}_{\text{eff}} = \sum_n \left[ \frac{c_n}{\Lambda^2} (\overline{q}_i t^a_{ij} \psi^a)(\chi_n q_j) + \text{h.c.} \right]$$

Assume parent's total width $\Gamma_\psi$ dominated by decays of the form $\psi \rightarrow jj\chi_n$.

Density of states decreases with $n$.

**Branching fractions** of $\psi$ to the different $\chi_n$ controlled by $\Delta m$, $\delta$, and $\gamma$.

Density of states increases with $n$. 

**Coupling strength** increases with $n$ for $\gamma > 0$...

...but phase space always decreases with $n$. 

Coupling strengths:

- $\gamma = -2$
- $\gamma = -1$
- $\gamma = 0$
- $\gamma = 1$
- $\gamma = 2$
DDM Ensembles & Kinematic Distributions

- Evidence of a DDM ensemble can be ascertained from characteristic features imprinted on the kinematic distributions of these SM particles.

For example, in the scenarios we're considering here, the (normalized) dijet invariant-mass distribution is given by

\[
\frac{1}{\Gamma_{\psi}} \frac{d\Gamma_{\psi}}{dm_{jj}} = \sum_{n=0}^{n_{\text{max}}} \left( \frac{1}{\Gamma_{\psi n}} \frac{d\Gamma_{\psi n}}{dm_{jj}} \times BR_{\psi n} \right)
\]
Increasing $\delta$

Two Characteristic Signatures:

1. **Multiple distinguishable peaks**
   
   Large $\delta$, $\Delta m$: individual contributions from two or more of the $\chi_n$ can be resolved.

2. **The Collective Bell**
   
   Small $\delta$, $\Delta m$: Individual peaks cannot be distinguished, mass edge “lost,” $m_{jj}$ distribution assumes a characteristic shape.
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   Small $\delta, \Delta m$: Individual peaks cannot be distinguished, mass edge “lost,” $m_{jj}$ distribution assumes a characteristic shape.
But the **REAL** question is...

**How well can we distinguish these features in practice?**

In other words: to what degree are the characteristic kinematic distributions to which DDM ensembles give rise truly **distinctive**, in the sense that they cannot be reproduced by **any** traditional DM model?

**The Procedure:**

- Survey over traditional DM models with different DM-candidate masses $m_\chi$ and coupling structures.
- Divide the into bins with width determined by the invariant-mass resolution $\Delta m_{jj}$ of the detector (dominated by jet-energy resolution $\Delta E_j$).
- For each value of $m_\chi$ in the survey, define a $\chi^2$ statistic $\chi^2(m_\chi)$ to quantify the degree to which the two resulting $m_{jj}$ distributions differ.

$$\chi^2(m_\chi) = \sum_k \frac{(X_k - E_k(m_\chi))^2}{\sigma_{2k}^2}$$

$$\chi^2_{\text{min}} = \min_{m_\chi} \left\{ \chi^2(m_\chi) \right\}$$

- The **minimum** $\chi^2$ value from among these represents the degree to which a DDM ensemble can be distinguished from **any** traditional DM candidate.
Distinguishing DDM Ensembles: Results

Results for $N_e = 1000$ signal events (e.g., $pp\rightarrow\psi\psi$ for TeV-scale parent, $L_{\text{int}} < 30$ fb$^{-1}$)

$\Delta m = 50$ GeV

$\Delta m = 200$ GeV

$\Delta m = 500$ GeV

Significance:
Distinguishing DDM Ensembles: Results

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BRs to all $\chi_n$ with $n > 1$ suppressed: lightest constituent dominates the width of $\psi$. 
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Density of states large enough to overcome $\gamma$ suppression for small $\delta$.

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BRs to all $\chi_n$ with $n > 1$ suppressed: lightest constituent dominates the width of $\psi$.

Next-to-lightest constituent $\chi_1$ dominates the width of $\psi$.

Density of states large enough to overcome $\gamma$ suppression for small $\delta$. 

Significance:

$1\sigma$ $2\sigma$ $3\sigma$ $4\sigma$ $5\sigma$
Distinguishing DDM Ensembles: Results

Results for $N_e = 1000$ signal events (e.g., $pp \to \psi \psi$ for TeV-scale parent, $L_{\text{int}} < 30 \text{ fb}^{-1}$)

$\Delta m = 50 \text{ GeV}$

$\Delta m = 200 \text{ GeV}$

$\Delta m = 500 \text{ GeV}$

BRs to all $\chi_n$ with $n > 1$ suppressed: lightest constituent dominates the width of $\psi$.

Next-to-lightest constituent $\chi_1$ dominates the width of $\psi$.

BR$(\psi \to jj\chi_0) \approx$ BR$(\psi \to jj\chi_1)$: two distinct $m_{jj}$ peaks.

Density of states large enough to overcome $\gamma$ suppression for small $\delta$. 

Significance:
Distinguishing DDM Ensembles: Results

Results for $N_e = 1000$ signal events (e.g., $pp \rightarrow \psi \psi$ for TeV-scale parent, $L_{\text{int}} < 30$ fb$^{-1}$)

\[ \Delta m = 50 \text{ GeV} \quad \Delta m = 200 \text{ GeV} \quad \Delta m = 500 \text{ GeV} \]

The Main Message:
DDM ensembles can be distinguished from traditional DM candidates at the 5s level throughout a substantial region of parameter space.

Density of states large enough to overcome $\gamma$ suppression for small $\delta$.

BRs to all $\chi_n$ with $n > 1$ suppressed: lightest constituent dominates the width of $\psi$.

Next-to-lightest constituent $\chi_1$ dominates the width of $\psi$.

BR($\psi \rightarrow jj\chi_0$) $\approx$ BR($\psi \rightarrow jj\chi_1$): two distinct $m_{jj}$ peaks.

Significance:
1σ, 2σ, 3σ, 4σ, 5σ
Distinguishing DDM Ensembles: Results

Results for $N_e = 1000$ signal events (e.g., $pp \rightarrow \psi \psi$ for TeV-scale parent, $L_{\text{int}} < 30$ fb$^{-1}$)

$\gamma = -1$  
$\gamma = 0$  
$\gamma = 1$

Large number of states accessible for small $\Delta m$, $\delta$
Distinguishing DDM Ensembles: Results

Results for $N_e = 1000$ signal events (e.g., $pp \rightarrow \psi\psi$ for TeV-scale parent, $L_{\text{int}} < 30$ fb$^{-1}$)

$$\gamma = -1 \quad \gamma = 0 \quad \gamma = 1$$

Large number of states accessible for small $\Delta m, \delta$

$\text{BR}(\psi \rightarrow jj\chi_0) \approx \text{BR}(\psi \rightarrow jj\chi_1)$: two distinct $m_{jj}$ peaks.
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Results for $N_e = 1000$ signal events (e.g., $pp \to \psi\psi$ for TeV-scale parent, $L_{\text{int}} < 30 \text{ fb}^{-1}$)

$\gamma = -1$

$\gamma = 0$

$\gamma = 1$

$\Delta m$

$\delta$

$\Delta m$

$\delta$

$\Delta m$

$\delta$

Large number of states accessible for small $\Delta m, \delta$

$\text{BR}(\psi \to jj\chi_0) \approx \text{BR}(\psi \to jj\chi_1)$: two distinct $m_{jj}$ peaks.

Only $\chi_0$ and $\chi_1$ kinematically accessible. One or the other dominates the width of $\psi$. 
Part II: Distinguishing DDM at Direct-Detection Experiments
Direct-detection experiments offer another possible method for distinguishing DDM ensembles from traditional DM candidates.

After the initial observation an excess of signal events at such an experiment, the shape of the recoil-energy spectrum associated with those events can provide additional information about the properties of the DM candidate.

A number of factors impact the shape of the recoil-energy spectrum in a generic dark-matter scenario. Particle physics, astrophysics, and cosmology all play an important role.
Direct Detection of DDM

In this talk, I'll adopt the following standard assumptions about the particles in the DM halo as a definition of the “standard picture” of DM:

- Total local DM energy density: $\rho_{\text{loc}}^{\text{tot}} \approx 0.3 \text{ GeV/cm}^3$.
- Maxwellian distribution of halo velocities for all $\chi_j$.
- Local circular velocity $v_0 \approx 220 \text{ km/s}$, galactic escape velocity $v_e \approx 540 \text{ km/s}$.
- Woods-Saxon form factor.
- Spin-independent (SI) scattering dominates.
- Isospin conservation: $f_{pj} = f_{nj}$.
- Local DM abundance $\propto$ global DM abundance: $\rho_{\text{loc}}^{\text{tot}} / \rho_{\text{loc}}^{\text{tot}} \approx \Omega_j / \Omega_{\text{tot}}$.

Departures from this standard picture (isospin violation, non-standard velocity distributions, etc.) can have important experimental consequences.

Here, we examine the consequences of replacing a traditional DM candidate with a DDM ensemble, with all other things held fixed.
Recoil-Energy Spectra: Traditional DM

- Let's begin by reviewing the result for the spin-independent scattering of a traditional DM candidate $\chi$ off a an atomic nucleus $N$ with mass $m_N$.
- Recoil rate exponentially suppressed for $E_R > 2m_\chi^2m_Nv_0^2/(m_\chi+m_N)^2$

Target material: Xe
Normalization: $\sigma_{N\chi} = 1$ pb

Two Mass Regimes:

**Low-mass regime:** $m_\chi \leq 20 - 30$ GeV
- Spectrum sharply peaked at low $E_R$ due to velocity distribution. Shape quite sensitive to $m_\chi$.

**High-mass regime:** $m_\chi \geq 20 - 30$ GeV
- Broad spectrum. Shape not particularly sensitive to $m_\chi$.
DDM Ensembles and Particle Physics

- Cross-sections depend on effective couplings between the $\chi_j$ and nuclei.
- Both elastic and inelastic scattering can in principle contribute significantly to the total SI scattering rate for a DDM ensemble.
- In this talk, I'll focus on elastic scattering: $\chi_j N \rightarrow \chi_j N$.
- For concreteness, I'll focus on the case where the couplings between the $\chi_j$ and nucleons scale like:

$$f_{n,j} = f_{n0} \left( \frac{m_j}{m_0} \right)^\beta$$

$$\sigma_{n,j}^{(SI)} = \frac{4\mu_{n,j}^2}{\pi} f_{n,j}^2$$

- However, note that inelastic scattering has special significance within the DDM framework:
  - Possibility of downscattering ($m_k < m_j$) as well as upscattering ($m_k > m_j$) within a DDM ensemble.
  - Scattering rates for $\chi_j N \rightarrow \chi_k N$ place lower bounds on rates for decays of the form $\chi_j \rightarrow \chi_k + \text{[SM fields]}$ and hence bounds on the lifetimes of the $\chi_j$. 

Elastic Scattering

Inelastic Scattering
• In contrast to the collider analysis presented in the first half of this talk, direct detection involves a **cosmological population** of DM particles, and thus aspects of DDM cosmology.

• Recall that the cosmology of a given DDM ensemble is primarily characterized by two parameters: $\eta$ and $\Omega_{\text{tot}}$.

• For concreteness, consider the case where $m_j = m_0 + n^\delta \Delta m$ and the present-day abundances $\Omega_j$ scale like:

\[
\eta = 1 - \frac{\Omega_0}{\Omega_{\text{tot}}}
\]

\[
\Omega_j = \Omega_0 \left( \frac{m_j}{m_0} \right)^{\alpha}
\]

\[
\Delta m/m_0 = 1
\]

\[
\Delta m/m_0 = 10^{-3}
\]

$\eta \sim \mathcal{O}(1)$: the full ensemble contributes significantly to $\Omega_{\text{tot}}$. 

**Summary:**

- **For concreteness,** consider the case where $m_j = m_0 + n^\delta \Delta m$ and the present-day abundances $\Omega_j$ scale like:

\[
\Omega_j = \Omega_0 \left( \frac{m_j}{m_0} \right)^{\alpha}
\]

- **Cosmological population** of DM particles leads to **DDM Ensembles and Cosmology**.

- **Key parameters:** $\eta$ and $\Omega_{\text{tot}}$.

- **Graphical representation:**
  - $\eta$ as a function of $\alpha$ and $\delta$.
  - $\Delta m/m_0 = 1$ and $\Delta m/m_0 = 10^{-3}$.
Recoil-Energy Spectra: DDM

- **Distinctive features** emerge in the recoil-energy spectra of DDM models, especially when one or more of the $\chi_j$ are in the low-mass regime.

- As $m_0$ increases, more of the $\chi_j$ shift to the high-mass regime. Spectra increasingly resemble those of traditional DM candidates with $m_\chi \approx m_0$.

$$\Delta m = \{1, 10, 40, 100\} \text{ GeV}$$

$\chi_j$ with $m_\chi = \{10, 100\} \text{ GeV}$

- Xe target, Rate normalized to that of $\chi$ with $\sigma^{(SI)} = 10^{-9} \text{ pb}$

- $m_0 = \{30, 100\} \text{ GeV}$


Constraining Ensembles:

- Experimental limits constrain DDM models just as they constrain traditional DM models.
- A DDM ensemble has no well-defined mass or interaction cross-section: limits cannot be phrased as bounds on $m_\chi$ and $\sigma_\chi^{(SI)}$.
- Most stringent limits from XENON100 data.

Bound on $\chi_0$ $\sigma_{n0}^{(SI)}$ in DDM models:

$\sigma_{n0,\max}^{(SI)} [\text{cm}^{-2}]$:

Not appropriate for DDM
Consider the case in which a particular experiment, characterized by certain attributes including...

- Target material(s)
- Detection method
- Fiducial Volume
- Data-collection time
- Signal acceptance
- Recoil-energy window

...reports a statistically significant excess in the number of signal events.

**The Procedure (much like in our collider analysis):**

- Compare the recoil-energy spectrum for a given DDM ensemble to those of traditional DM candidates which yield the same total event rate at a given detector.

- Survey over traditional DM candidates with different $m_\chi$ and define a $\chi^2$ statistic for each $m_\chi$ to quantify the degree to which the corresponding recoil-energy spectrum differs from that associated with the DDM ensemble.

- The minimum $\chi^2_{\text{min}}$ of these quantifies the degree to which the DDM model can be distinguished from traditional DM candidates, under standard astrophysical assumptions.
As an example, consider a detector with similar attributes to those anticipated for the next generation of noble-liquid experiments (XENON1T, LUX, PANDA-X, et al.). In particular, we take:

- Liquid-xenon target
- Fiducial volume ~ 1000 kg
- Five live years of operation.
- Energy resolution similar to XENON100
- Acceptance window: $8.4\text{ keV} < E_R < 44.6\text{ keV}$

**Background Contribution**

- $N_e \sim 1000$ total signal events observed (consistent with most stringent current limits from XENON100).
- Background $dR/dE_R$ spectrum essentially flat

\[
\frac{dR}{dE_R} \approx 7 \times 10^{-9} \text{ day}^{-1} \text{ kg}^{-1} \text{ keV}^{-1} \text{ nr}^{-1}
\]
Distinguishing DDM Ensembles: Results

Graphs showing the significance of signal versus mass and mass difference for different parameters. The significance is color-coded from yellow (1σ) to red (5σ).
Distinguishing DDM Ensembles: Results

All $\chi_n$ in high-mass regime: little difference between their $dR/dE_R$ contributions
Distinguishing DDM Ensembles: Results

\( \chi_0 \) contributes mostly at \( E_R < E_{R}^{\text{min}} \),
all other \( \chi_j \) in high-mass regime

All \( \chi_n \) in high-mass regime: little difference
between their \( dR/dE_R \) contributions
Distinguishing DDM Ensembles: Results

\( \chi_0 \) contributes mostly at \( E_R < E_R^{\text{min}} \), all other \( \chi_j \) in high-mass regime.

All \( \chi_n \) in high-mass regime: little difference between their \( dR/dE_R \) contributions.

\( \chi_0 \) in low-mass regime, all \( \chi_j \) with \( j \geq 1 \) in high-mass regime: kink in \( dR/dE_R \) spectrum.
Distinguishing DDM Ensembles: Results

\( \chi_0 \) contributes mostly at \( E_R < E_R^{\text{min}} \), all other \( \chi_j \) in high-mass regime:

- Little difference between their \( dR/dE_R \) contributions.

All \( \chi_n \) in high-mass regime:

- \( \chi_0 \) contributes mostly at \( E_R < E_R^{\text{min}} \).
- \( \chi_j \) with \( j \geq 1 \) in high-mass regime:
  - Kink in \( dR/dE_R \) spectrum.

\( \chi_0 \) in low-mass regime, all \( \chi_j \) with \( j \geq 1 \) in high-mass regime:

- Only \( \chi_0 \) contributes perceptible to overall rate: looks like regular low-mass DM.

\( \alpha = -1.5 \)
\( \beta = -1.5 \)
\( \delta = 1 \)
Distinguishing DDM Ensembles: Results

\( \chi_0 \) contributes mostly at \( E_R < E_R^{\text{min}} \), all other \( \chi_j \) in high-mass regime

All \( \chi_n \) in high-mass regime: little difference between their \( dR/dE_R \) contributions

\( \chi_0 \) in low-mass regime, all \( \chi_j \) with \( j \geq 1 \) in high-mass regime: kink in \( dR/dE_R \) spectrum

Only \( \chi_0 \) contributes perceptible to overall rate: looks like regular low-mass DM

Multiple \( \chi_j \) in low-mass region: distinctive \( dR/dE_R \) spectra
The upshot:

In a variety of situations, it should be possible to distinguish characteristic features to which DDM ensembles give rise at the next generation of direct-detection experiments.

- The best prospects are obtained in cases where multiple $\chi_j$ are in the low-mass regime: $m_j \lesssim 30$ GeV.
- A $5\sigma$ significance of differentiation is also possible in cases in which only $\chi_0$ is in the low-mass regime and a kink in the spectrum can be resolved.

**CAUTION** Discrepancies in recoil-energy spectra from standard expectations can arise due to several other factors as well (complicated halo-velocity distribution, velocity-dependent interactions, etc.). Care should be taken in interpreting such discrepancies in the context of any particular model.

However,

By comparing/correlating signals from **multiple experiments** it should be possible to distinguish between a DDM interpretation and many of these alternative possibilities.
Summary

• Dynamical dark matter (DDM) is a new framework for addressing the dark-matter question in which stability is replaced by a balancing between lifetimes and abundances across a vast ensemble of particles $\chi_n$ which collectively account for $\Omega_{\text{CDM}}$.

• DDM scenarios give rise to a variety of distinctive experimental signatures which can be used to differentiate DDM ensembles from traditional DM candidates.

• DDM ensembles can give rise to distinctive features in the kinematic distributions of SM fields produced in conjunction with the $\chi_n$ via the decays of other heavy particles.

• DDM ensembles can also leave imprints on the recoil-energy spectra observed at direct-detection experiments.

Other possibilities? Indirect detection?

Indeed, the full range of phenomenological consequences of the DDM framework is just beginning to be explored!
Extra Slides
In most dark-matter models, the dark sector consists of one stable dark-matter candidate $\chi$ (or a few such particles). Such a dark-matter candidate must therefore...

- account for essentially the entire dark-matter relic abundance observed by WMAP: $\Omega_{\chi} \approx \Omega_{\text{CDM}} \approx 0.23$.
- Respect observational limits on the decays of long lived relics (from BBN, CMB data, the diffuse XRB, etc.) which require that $\chi$ to be extremely stable:

$$\tau_\chi \gtrsim 10^{26} \text{ s}$$

(Age of universe: only $\sim 10^{17}$ s)

**Consequences**

- Such “hyperstability” is the **only** way in which a single DM candidate can satisfy the competing constraints on its abundance and lifetime.
- The resulting theory is essentially “frozen in time”: $\Omega_{\text{CDM}}$ changes only due to Hubble expansion, etc.
Nothing special about the present time: DM decays before, during, and after the current epoch. The DM abundance and composition are constantly evolving!
Characterizing DDM Ensembles

• The cosmology of DDM models is principally described in terms of three fundamental (time-dependent) quantities:

1. Total relic abundance:

\[ \Omega_{\text{tot}}(t) = \sum_{i=0}^{N} \Omega_i(t) \]

2. Distribution of that abundance:

(One useful measure)

\[ \eta(t) \equiv 1 - \frac{\Omega_0}{\Omega_{\text{tot}}} \]

where

\[ \Omega_0 \equiv \max \{ \Omega_i \} \]

The interpretation:

\[ 0 \leq \eta \leq 1 \]

\[ \begin{cases} \eta = 0 & \text{One dominant component (standard picture)} \\ \eta > 0 & \text{Quantifies departure from traditional DM} \end{cases} \]

3. Effective equation of state:

\[ p = w_{\text{eff}} \rho_{\text{tot}} \]

\[ w_{\text{eff}}(t) = - \left( \frac{1}{3H} \frac{d\rho_{\text{tot}}}{dt} + 1 \right) \]

Not always \( w = 0! \)
Unlike traditional dark-matter candidates, a DDM ensemble has no well-defined mass, decay width, or set of scattering cross-sections.

The natural parameters which describe such a dark-matter candidate are those which describe the internal structure of the ensemble itself and describe how quantities such as the constituent-particle masses, abundances, decay widths, and cross-sections scale with respect to one another across the ensemble as a whole.

For example:

\[ \Omega(\Gamma) = A \left( \frac{\Gamma}{\Gamma_0} \right)^\alpha \]

\[ n_\Gamma(\Gamma) = B \left( \frac{\Gamma}{\Gamma_0} \right)^\beta \]

Density of states per unit width \( \Gamma \)

The properties of the ensemble are naturally expressed in terms of the coefficients \( A \) and \( B \) and the scaling exponents \( \alpha \) and \( \beta \).

\[ e.g., \text{ if we take: } \Omega_i(t) \approx \Omega_i \Theta(\tau_i - t) \]

\[ \sum_i \rightarrow \int n_\tau(\tau)d\tau \quad \text{with} \quad n_\tau = \Gamma^2 n_\Gamma \]

We obtain the general result:

\[ \frac{d\Omega_{\text{tot}}(t)}{dt} \approx -\sum_i \Omega_i \delta(\tau_i - t) \approx -AB\Gamma_0^2(\Gamma_0 t)^{-\alpha-\beta-2} \]