Universal contributions to scalar masses from 5d supergravity

Emilian Dudas (CERN-TH and Ecole Polytechnique)

Melbourne, July 2, 2012

Work in progress with

Gero GERSDORFF
Motivations:

SUSY breaking and transmission is the main open problem in low-energy SUSY models:

- couplings between matter fields $Q$ and spurion $X$ determine SUSY soft breaking terms.

- they are highly dependent on the UV physics: string theory, geometry of internal space, nature of spurion.

- without further assumptions, difficult to say more about soft terms.

Couplings matter-spurion can be of several types:

- present at the supergravity tree-level (gravity mediation)
- generated by low-energy dynamics (gauge mediation)
- generated by exchange of massive string/KK modes
We will use a simple (and popular) framework to investigate the effects of massive spectrum on matter/spurion couplings: 

5d AdS gauged supergravity theory inspired by the AdS/CFT correspondence, with:

- 5d N=1 SUSY gravity multiplet, containing 4d graviton + KK resonances : 4d CFT sector
- 5d hypermultiplets localized by mass terms/gauging, which contains charged SM matter and the spurion X
  - additional 5d vector multiplets (gauge fields): SM gauge group

Other fields/multiplets could be added: hypers (complex structure Moduli), vectors (Kahler moduli, additional gauge groups).
This will be a local SUSY extension of RS type models

Popular brane-world model: **Randall-Sundrum model**

There is a 5d-4d holographic dictionary:
- states localized on the TeV (IR) brane: composite.
- states localized on the Planck (UV) brane: elementary.
- Different matter localization

Couplings to CFT operators of different dimensions

- For flat internal space, the exchange of the KK gravity multiplet only generates dim. 8 higher-derivative effective operators, not relevant for low-energy physics.

- Due to the curvature of internal space/gauging of SUGRA, fermions and scalars acquire couplings with less derivatives: (ex. minimal couplings to graviphoton).

  dim. 6 operators in the Kahler potential.

- the summation over the KK modes can be done exactly, due to the simple properties of Green functions in AdS.

- neglecting KK contributions to the effective action is inconsistent.
Outline

✗ Integrating out the heavy mesons - the role of the R-current
✗ A 5D model - supergravity with hypermultiplets
✗ The effective Kahler potential at low energy and implications for soft terms
✗ Conclusions
Integrating out mesons

✗ CFT spectrum of operators not only contains spin-0 mesons

✗ Higher spin operators will contribute to effective action

✗ In particular: currents (spin one operators)

✗ Corresponding to conserved or nonconserved global symmetries

\[ \langle J_{\mu}^{UV} J_{\nu}^{UV} \rangle = g_{\mu\nu} \sum \frac{F_n^2}{p^2 + m_n^2} \]

\[ \mathcal{L}_{eff} \supset \left( J_{\mu}^{IR} \right)^2 \xrightarrow{} K \supset \sum_i \alpha_{ij}(\bar{\Phi}_i \Phi_i)(\bar{\Phi}_j \Phi_j) \]
Observations

✗ The masses of these mesons are determined by the mass gap of the confining theory, much less than Planck scale

\[ K \supset \sum \alpha_{ij}(\bar{\Phi}_i \Phi_i)(\bar{\Phi}_j \Phi_j) \]

Potentially: \[\alpha_{ij} \gg \frac{1}{M_{Pl}^2}\]

✗ These currents are very model dependent (global flavor group G)

✗ To generate soft terms, Goldstino field \( X \) must transform under G

✗ SCFT’s such as SQCD in conformal window possess a global abelian R symmetry under which ALL fields are charged

\[ q_R = \frac{2}{3} \Delta \]

Generates IRREDUCIBLE contribution to soft terms (→ this talk)
A Model in 5 dimensions

✗ AdS/CFT: Five dimensional SUGRA provides a simple computable framework for strongly coupled SUSY gauge theories.

\[ ds^2 = \frac{1}{(kz)^2} (dx^2 + dz^2) \]

5d SUGRA + matter (hypermultiplets)

UV brane (4d gravity)

IR brane (confinement)

✗ Resulting KK Lagrangian describes mesons and their interactions
Field content of 5d (minimal) SUGRA (8+8 d.o.f.):

- 5d metric $h_{MN}$ (5 d.o.f.)
- Graviphoton $A_M$ (3 d.o.f.)
- Gravitino $\psi^i_M$ (8 d.o.f.)

4d SUGRA, 2+2 d.o.f.

- 4d metric $h_{\mu\nu}$ (2 d.o.f.)
- Gravitino $\psi^1_\mu$ (2 d.o.f.)
- Radion, 2+2 d.o.f.
  - $x h_{55} + i A_5$ (2 d.o.f.)
  - $x \psi^2_5$ (2 d.o.f.)

$\times$ Integrate out the heavy KK modes in presence of 5d sources (5d energy-momentum tensor, 5d R-current)

$\times$ Properly subtract zero modes

$\times$ Extract quartic Kahler potential from 4-fermion terms (or, alternatively, from 2-derivative 4-scalar terms)
Hypermultiplets

- In 5d, minimal supersymmetry is $N=2$.
- Matter comes in hypermultiplets = two chiral multiplets $\Phi, \Phi_c$.
- Boundary conditions remove 1/2 of supersymmetries, e.g. $\Phi_c = 0$.
- Zero modes are chiral $\Phi(x, z) = \phi(x) z^{3/2-c}$.

$$W = \Phi_1 \Phi_2 \Phi_3$$
- The gauging gives rise to a negative cosmological constant, leading to an AdS5 vacuum with curvature related to the graviphoton coupling constant $g_R$

$$g_R^2 = \frac{3 k^2}{8 M^3}.$$ 

- The gauging also related 5d mass terms and 5d R charges:

**Fermions:**

$$m_{\Psi_i} = c_i k, \quad q_{\Psi_i}^R = -\frac{2}{3} c_i.$$ 

**Scalars:**

$$m_{\Phi_i}^2 = \left( c_i^2 \pm c_i - \frac{15}{4} \right) k^2, \quad q_{\Phi_i}^R = 1 \mp \frac{2}{3} c_i.$$
fermionic couplings

The Lagrangian containing matter fermions reads

\[ \mathcal{L} = -i \bar{\Psi} \gamma^M D_M \Psi - i m_\Psi \bar{\Psi} \Psi - \sqrt{\frac{3}{128 M^3}} \bar{\Psi} \gamma^{AB} \Psi F_{AB} \frac{1}{128 M^3} (\bar{\Psi} \gamma_{AB} \Psi)^2 \]

\[ m_\Psi = c k \]

\[ k^2 = \frac{8 M^3 g_R^2}{3} \]

\[ D_M = \partial_M + ig_R \left( -\frac{2}{3} \right) A_M + \Gamma_M \]

- 5d R charge proportional to mass
- dipole interaction
- 5d four-fermion term
Effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = + \frac{1}{8 M_{Pl}^2} \int_{z_0}^{z_1} dz \, z^3 \left[ \Theta_{\mu\nu}(z) - \Omega_2(z) \Theta_{\mu\nu}(z_1) \right]^2 \]

\[ - \frac{1}{32 M_{Pl}^2} \int_{z_0}^{z_1} dz \, z^3 \left[ \Theta_{tr}(z) - \Omega_2(z) \Theta_{tr}(z_1) \right]^2 \]

\[ + \frac{1}{4 M_{Pl}^2} \left( \int_{z_0}^{z_1} dz \, z \left[ \Theta_{\mu 5}(z) \right]^2 - \frac{2}{z_1^2 - z_0^2} \left[ \int_{z_0}^{z_1} dz \, z \Theta_{\mu 5}(z) \right]^2 \right) \]

\[ + \frac{1}{12 M_{Pl}^2} \int_{z_0}^{z_1} dz \, z^{-1} \left[ \Theta_{55}(z) - \Omega_0(z) \Theta_{55}(z_1) \right]^2 \]

(1)

\[ \Theta_{\mu\nu}(z, x) = \int_{z_0}^{z} dz' \, z'^{-3} \left[ T_{\mu\nu}(z', x) - \frac{1}{4} \eta_{\mu\nu} T_{\rho\rho}(z', x) \right] \]

\[ \Theta_{tr}(z, x) = \int_{z_0}^{z} dz' \, z'^{-3} T_{\rho\rho}(z', x) \]

\[ \Theta_{55}(z, x) = \int_{z_0}^{z} dz' \, z'^{-1} \left[ T_{55}(x, z') - \frac{1}{2} T_{\rho\rho}(x, z') \right] \]

\[ \Theta_{\mu 5}(z, x) = \int_{z_0}^{z} dz' \, z'^{-2} T_{\mu 5}(z', x) \]

Similar Lagrangian can be written for the contribution of 5d gauge fields

\[ \Omega_0 = \frac{z^2 - z_0^2}{z_1^2 - z_0^2} \]

\[ \Omega_2 = \frac{z^{-2} - z_0^{-2}}{z_1^{-2} - z_0^{-2}} \]

Cabrera, Gersdorff, Quirós '11
For 4-fermion terms just two diagrams:

✗ 5d 4-fermion term
✗ graviphoton exchange

\[ K_{\text{eff}} = \left( \frac{1}{8M_{\text{Pl}}^2} + \frac{1}{6} \alpha_{ij} \right) \Phi_i \Phi_i \Phi_j \Phi_j \]

always > 0 !

\[ \alpha_{ij} = -\frac{1}{8M_{\text{Pl}}^2} \frac{(3 - 2c_i)(3 - 2c_j)}{(4 - 2c_i - 2c_j)} + \frac{1}{8M_{\text{Pl}}^2 \epsilon^2} \frac{(1 - 2c_i)(1 - 2c_j)}{(4 - 2c_i - 2c_j)} \frac{(1 - \epsilon^{3-2c_i})(1 - \epsilon^{3-2c_j})}{(1 - \epsilon^{1-2c_i})(1 - \epsilon^{1-2c_j})} \]

\[ \epsilon = \frac{z_0}{z_1} \]
 Scalars

✗ We can also reconstruct the quartic Kahler from dimension-6, four scalar, 2-derivative terms

✗ There are now more contributions

✗ These contributions lead to the same Kahler potential!

\[ K_{eff} = \left( \frac{1}{8M_{Pl}^2} + \frac{1}{6}\alpha_{ij} \right) \Phi_i \Phi_i \Phi_j \Phi_j \]
Remark on N=2 sigma models

✗ The N=2 hypermultiplets parametrize a nontrivial sigma model

✗ Supersymmetry fixes the curvature of these manifolds to be negative and related to the (5d) Planck mass

U model
\[
\frac{U(n_H,2)}{U(n_H) \times U(2)}
\]
compactify
\[
\frac{U(n_H,1)}{U(n_H) \times U(1)}
\]

S model
\[
\frac{USp(2n_H,2)}{USp(2n_H) \times USp(2)}
\]

\[
K = -p \log \left( 1 - p^{-1} \Phi_i \Phi_i \right)
\]

✗ U model: \( p = 1 \)
✗ S model: \( p = 2 \)

✗ This leads to tachyonic soft masses
Scalar masses:

- The Kahler potential we obtain after the reduction to 4d is

\[ K = -p \ln \left( 1 - \frac{1}{p} \sum_{i=1}^{n} |\Phi_i|^2 \right) + d_p \alpha_{ij} |\Phi_i|^2 |\Phi_j|^2 , \]

where \( p=1, \ d_p = 2/3 \) for unitary spaces \( U(2,n)/U(2) \times U(n) \) and \( p=2, d_p=1/6 \) for symplectic spaces \( USp(2,2n)/USp(2) \times USp(2n) \).

- The first term in the Kahler, parameterizing \( U(1,n)/U(1) \times U(n) \), describes the truncation of both quaternionic spaces to \( N=1 \) in 4d.

- The second term comes from integrating the heavy states.
Scalar soft masses for matter fields are computed starting from

\[ m_{a\bar{b}}^2 = m_{3/2}^2 \left( G_{a\bar{b}} - G_\alpha R_{a\bar{b}\alpha\bar{\beta}} G^{\bar{\beta}} \right) , \]

where \( a, b = \) matter indices, \( \alpha, \beta \) are spurion indices

and

\[ G^\alpha G_{\alpha\bar{\beta}} G^{\bar{\beta}} = 3. \]

is the condition for zero cosmological constant in 4d.

The first (geometric) term in the Kahler is an Einstein space

\[ R_{ij\bar{k}\bar{l}} = \frac{1}{p} \left( G_{ij} G_{k\bar{l}} + G_{i\bar{l}} G_{k\bar{j}} \right) , \]
This geometric part contributes to scalar masses

\[(m_a^2)_{\alpha i j = 0} = m_{3/2}^2 \left( 1 - \frac{3}{p} \right).\]

The second term, dependent on the localization of the fields, is

\[
(m_a^2)_{\alpha i j} = -\frac{4\alpha_{a\beta}}{3} |F_\beta|^2 = -\frac{4m_{3/2}^2}{3} \alpha_{a\beta} |G_\beta|^2, \text{ unitary case } p = 1,
\]

\[
(m_a^2)_{\alpha i j} = -\frac{\alpha_{a\beta}}{3} |F_\beta|^2 = -\frac{m_{3/2}^2}{3} \alpha_{a\beta} |G_\beta|^2, \text{ symplectic case } p = 2,
\]

Adding both terms, we find

\[
m_a^2 = -2m_{3/2}^2 \left( 1 + \frac{2}{3} \alpha_{a\beta} |G_\beta|^2 \right) \text{ unitary case } p = 1,
\]

\[
m_a^2 = -\frac{1}{2} m_{3/2}^2 \left( 1 + \frac{2}{3} \alpha_{a\beta} |G_\beta|^2 \right) \text{ symplectic case } p = 2.
\]
- For the sequestered case where the matter field and the spurion are sharply localized at opposite boundaries to the internal space $S^1/Z_2$, we find vanishing scalar masses!

- This result is consistent with sequestering arguments.

- For all other localization parameters, scalar masses are tachyonic!

Comments:

- We don’t know the holographic interpretation of this result.

- This implies strong constraints on model building: presence of localized Kahler operators, contributions from other type of spurions: radion, bulk vector multiplets, etc.
Conclusions

✗ Have considered 5D supersymmetric models of flavor in a slice of AdS5 spacetime (SUSY-RS with potentially large IR scale).

✗ Integrating out 5D supergravity yields new contributions to the Kahler potential of chiral zero modes.

✗ These irreducible contributions give tachyonic scalars.

✗ Via AdS/CFT: dual to the contributions of mesons excited by the R-current (FZ multiplet of currents).

✗ Possible additional contributions include:
  - Other global (non - R) currents (read: 5d gauge multiplets)
  - Brane localized Kahler potentials
  - Radion mediation
  - Higher order terms in Kahler potential
Some other comments:

- $U(1)_R$ gauging generates effective operators of the type

$$\left(\frac{1}{\Lambda_{IR}^2}\right)(H^\dagger D_\mu H)^2.$$  

They break custodial symmetry.

Limits on the $\rho$ parameter $\Lambda_{IR} > 10$ TeV

- Although we discussed explicitly only the AdS case, we expect the massive KK exchange contributions to be important in other warped compactifications.